# Z-transform, System function and Stability

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| Objective: To know how to plot the frequency response of a system based on its z-transform system function.  To know the relationship between the system function and its impulse response.  To use the pole-zero plot to determine the stability of the digital system. |

## Background information

The z-transform, an important tool for discrete-time signal and system analysis, can be viewed as the discrete-time form of the Laplace transform. It provides another domain in which signals and systems can be examined. The z-transform is very closely related to the discrete-time Fourier transform and finds its principal use in the analysis and manipulation of linear, constant coefficient difference equations.

The two-sided or bilateral z-transform is defined as the summation



The summation in the equation is a power series in the complex variable z-1, the coefficients of which are the samples of sequence x(n).

One of the applications of z-transform is the determination of system function of a discrete-time system. System function may be defined as the ratio of the z-transform of the output of the system to the z-transform of the input to the system. It can easily be shown that the z-transform of the impulse response of a system yields the system function.

X(z)

Y(z)

x(n)

y(n)

System

H(z)

h(n)



Much information about the system may be extracted from the system function — such as the frequency response of the system. The relationship between traditional Fourier transform and z-transform can be summarised by a simple relation: . In another words, to find the frequency response of a system, we merely need to substitute all the z-terms in the expression with ejω, where  is the normalised angular frequency of interest with fs being the sampling frequency of the system.

## Procedure 1: Frequency response from the system function

1. Consider the system function H1(z) defined as



The frequency response of the system may be obtained by replacing z with ejω and then evaluating the function H1 at different values of ω. Hence, we have



The plotting of the magnitude and phase angle of H1 against normalised angular frequency, ω, would yield the amplitude and phase response of the system respectively.

1. To obtain the frequency response of H1 for 0 ≤ ω ≤ π, invoke the following MATLAB commands:

>>w=[0:0.01:pi];

>>N=1+3\*exp(-j\*w)+3\*exp(-j\*2\*w)+exp(-j\*3\*w);

>>D=1+0.5\*exp(-j\*w)+0.3\*exp(-j\*2\*w)+0.1\*exp(-j\*3\*w);

>>H1=N./D;

1. To plot the frequency response:

>>subplot(2,1,1),plot(w,abs(H1));

>> title('Magnitude Response');xlabel('w');ylabel('|H1|');

>>subplot(2,1,2),plot(w,angle(H1));

>> title('Phase Response');xlabel('w');ylabel('Phase Angle of H1');

Record your results in Figure 7–1.



Figure 7–1

1. Consider another system function:



Plot the magnitude and phase response of the system with the following commands:

>>w=[0:0.01:pi];

>>H2=1-0.5\*exp(-j\*w);

>>subplot(211),plot(w,abs(H2));

>> title('Magnitude Response'); xlabel('w'); ylabel('|H2|');

>>subplot(212),plot(w,angle(H2));

>>title('Phase Response'); xlabel('w'); ylabel('Phase Angle of H2');

1. Repeat step 1-4 for the following system function:



What happen to the frequency response of the system when one of its coefficients changed its polarity?

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## Procedure 2: Frequency response using MATLAB function

1. Consider the system function as given in step 1-1. This function can be represented in MATLAB by storing the coefficients of the polynomials, B(z) and A(z), as follows:

>>B=[1 3 3 1];

>>A=[1 0.5 0.3 0.1];

1. MATLAB has a built-in function to compute for the frequency response of a system. Use the help utility to see a description of the function by typing:

>>help freqz

Use this function to compute and plot the frequency response of the above system:

>>w=[0:0.01:pi];

>>h1=freqz(B,A,w);

>>subplot(211),plot(w,abs(h1));

>>subplot(212),plot(w,angle(h1));

Compare your result with those recorded in Figure 7–1. Are they the same?

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1. Repeat steps 1-4 and 1-5 using the freqz function and verify your results is the same.

## Procedure 3: Impulse response

MATLAB has also provided another built-in function that helps us compute the output of a digital system for a given input sequence. A detailed description of the function is given by:

>>help filter

This **filter** command has been used previously for the difference equation.

1. The impulse response of the system given in step 1-1 can be found by the following commands:

>>B=[1 3 3 1];

>>A=[1 0.5 0.3 0.1];

>>y=filter(B,A,[1 0 0 0 0 0 0 0 0 0 0 0 0 0 0])

1. From the z-transform table, we can see that



If a system has system function given by:



determine the impulse response of the system mathematically.

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1. Verify your result in step 3-2 using MATLAB by:

>>B=[0 0.7071]; %numerator

>>A=[1 -1.4142 1]; %denominator

>>y=filter(B,A,[1 zeros(1,20)])

1. Plot the impulse response:

>>lineplot(y)

Is the impulse response a sinewave? Determine the frequency of the impulse response as a function of the sampling frequency.

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1. Repeat steps 3-3 to 3-5 for the following system:



Procedure 4: Stablility of the digital systems – Pole-zero diagram

4.1 Consider the system function H1(z) again



By analysis the position of the poles and zeros of the system function, we can determine whether the system is digital system is stable, marginally stable or unstable.

Poles, in particular, have great influence on system stability. Locations of the poles in the *z*-plane, determine the stability of the system. A system is stable if all of its poles are inside the unit circle. Unit circle refers to a circle with a radius of one unit centred at the origin of the *z*-plane. If one or more poles are on the unit circle, then the system is marginally stable.

>>B=[1 3 3 1];

>>A=[1 0.5 0.3 0.1];

>>roots(B)

>>roots(A)

>>zplane(B,A);

You will notice that there are 3 same values of -1 for the “zero” values in the numerator with the value -1 and 3 different values for the “pole” values in the denominator.

4.2 Consider the system function H4(z),



Plot the pole-zero diagram of the digital system. How many zero and pole values? Is the system stable?

>>B=[1 0 -1];

>>A=[1 -1.6 0.8];

>>roots(B)

>>roots(A)

>>zplane(B,A)

Exercise

For the following digital system, determine

(a) the frequency response

(b) the impulse response

